Matrix Theory and U-duality in Seven Dimensions

Moshe Rozali*

Theory Group, Department of Physics
University of Texas, Austin, Texas 78712

Abstract

We demonstrate the emergence of the U-duality group in compactification of Matrix theory on a 4-torus. The discussion involves non-trivial effects in strongly coupled 4+1 dimensional gauge theory, and highlights some interesting phenomena in the Matrix theory description of compactified M-theory.

1 Introduction

In the last few months there has emerged a candidate for the mysterious M-theory, formulated in the light cone gauge. The so called M(atrix) theory [1] passed several consistency conditions. It has the right structure of 11-dimensional supergravity, a membrane and a longtitudal fivebrane [7], with the right interactions [9, 3]. In [1, 6, 12, 3, 2] the compactification on tori was considered, with emphasis on T-duality, which is not manifest in the original formulation. Compactification on more complicated spaces was also considered [11]. Additional issues are addressed in [8].

One of the checks that the Matrix theory has yet to pass is the emergence of the expected U-duality groups in various dimensions. This was considered in [6] where the $SL(2, Z) \times SL(3, Z)$ symmetry in 8 dimensions was shown to hold, partially as a consequence of S-duality in $\mathcal{N}=4$ 3+1 dimensional YM theory. Subsequently the $SL(2, Z)$ U-duality in 9 dimension was studied [2].

In this letter we show the emergence of the expected U-duality group in 7 dimensions. This involves some non-trivial phenomena in strongly coupled SYM theories in 4+1 dimensions, which have a natural interpretation in M-theory. We also comment on a puzzle regarding gravitational anomalies on the Matrix model's base space.

2   U-Duality in Seven Dimensions

We consider compactifying M theory on a 4-torus down to 7 dimensions. The theory has 32 supersymmetries and is unique: the only possible multiplet is the gravity multiplet. The scalars in that multiplet parametrize the manifiold $SL(5, R)/SO(5)$. The conjectured U-duality group is $SL(5, Z)$. It is this structure that we want to discover in the Matrix theory formulation of M-theory.

As was elaborated in [12, 6], Matrix theory compactified on T^d is described by a d+1 dimensional field theory on the dual torus, with 16 supersymmetries. The resulting theory for the case at hand is then a 4+1 dimensional theory with $\mathcal{N}$=2 SUSY. The vector multiplet of this theory has one vector, five scalars and two Majorana spinors. The matrix theory has one vector multiplet in the adjoint of $U(N)$, with the scalars representing the 5 non-compact transverse directions. For later use we prefer to use the $\mathcal{N}$=1 language, in which the $\mathcal{N}$=2 vector multiplet decomposes as a vector multiplet (a vector, a spinor and a scalar) and an hypermultiplet (4 scalars and a spinor).

The action is the usual SYM action:

$$L = \frac{1}{4g^2} tr F_{\mu\nu} F^{\mu\nu} + ... \tag{2.1}$$

The parameters of this theory are identified by a straightforward application of the derivation in [6]. This simple calculation is performed in the appendix. We take the space 4-torus to be of sides L_a, and assume for simplicity it is rectangular. The gauge theory then has base space of sides S_a , and coupling constant g^2. Their values are given as:

$$S_a = \frac{2\pi (l_p)^3}{L_a R} \tag{2.2}$$

$$g^2 = \frac{(2\pi)^2 (l_p)^6}{R\, L_1 L_2 L_3 L_4}. \tag{2.3}$$

Where l_p denotes the 11-dimensional Planck length, and R is the length of the 11th dimension, to be taken to infinity together with N.

For general parameters L_i the effective coupling Ng^2 is not necessarily weak. Morover the SYM theory is not renormalizable, which would seem to lead to the problems of being not well-defined in the UV and being trivial in the IR. In what follows we find a weakly

coupled non-trivial fixed point theory equivalent to this gauge theory. This makes the dynamics more transparent, and in particular the $SL(5, Z)$ becomes manifest.

The gauge theory has a manifest $SL(4, Z)$ symmetry. It is the global symmetry mixing the parameters S_a, while keeping their product fixed. This doesn't change the gauge coupling (2.3). To extend this symmetry we look for another parameter of the gauge theory to combine with the parameters S_a. The only other parameter available is the gauge coupling itself. We note that $S_5 \equiv g^2$ has length dimension 1 in 4+1 dimensions, so it can naturally combine with the parameters S_a. Furthermore, since the coupling does not run, it can be treated as an additional parameter, independent of the parameters S_a.

We are therefore led to conjecture that the Matrix theory description of this compactification is a 5+1 dimensional theory, with base space that is a 5-torus. The sides of the base space torus are the dual 4-torus, combined with S_5. The $SL(5, Z)$, then, is just the geometric symmetry of the base space, mixing the parameters S_i (i=1,...,5) while keeping their product fixed.

The excitations corresponding to momentum states in the additional dimension are introduced in [5]. The gauge theory has instantons, which are point particles in 4+1 dimensions [1] , and have mass proportional to $\frac{1}{g^2}$ [2]. Analysis of the fermionic zero modes reveals that these instantons are an $\mathcal{N}$=2 tensor (or equivalently a vector) multiplet in the adjoint of $U(N)$ [3].

We propose the existence of a unique bound state at threshold of n of these instantons, for any n. We find support for this assumption in the next section, using known facts about type II string theory. As we take S_5 to be large , then, these states form a continuum, signalling a new dimension opening in the base space.

The origin of these states is also clear in M-theory: the instantons in the Matrix theory generally correspond to longtitudal fivebranes [12], so the new states in the Matrix theory that carry momentum of the additional dimension are just fivebranes completely wrapped around the 4-torus (and the lightcone direction). As an additional evidence we show in the appendix that the instanton mass coincides with the expected energy of a wrapped

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[1]By an instanton, in any number of dimensions, we mean a solution of the Euclidean YM equations with 4 transverse coordinates.

[2]This is their classical mass. As BPS saturated states they do not receive mass corrections.

[3]Since the base space of the theory is a torus, there is no non-compact collective coordinate corresponding to the instanton size. These are zero size instantons.

fivebrane. These states are included in the gauge theory, in agreement with the general approach to compactifications of the Matrix theory that requires including all the 0-branes of the compactified theory [1] [4] .

We see, then, that the wrapped fivebrane is included by the dynamics of the gauge theory, and does not have to be added by hand. Furthermore, T-duality is incorporated naturally. As the spacetime torus shrinks to zero, we know that the dynamics is more transparent in terms of the T-dual theory. In that theory the 4-branes are interchanged with the 0-branes. Therefore T-duality predicts that for small values of L_a the wrapped fivebrane should dominate the dynamics. This is indeed the case since this limit corresponds exactly to strong coupling in the gauge theory, a limit in which the instantons become light. In the 5+1 dimensional description of the theory there is no distinction between the original 0-branes and the wrapped 5-branes (instantons), thus T-duality is manifest in that description.

We conclude then that inclusion of the wrapped longtitudal fivebranes is accounted for by making the Matrix theory describing this compactification a 5+1 theory on a 5-torus. The U-duality group is then manifest classically in Matrix theory as a global symmetry acting on the parameters S_i [5].

in the next section we study the strong coupling behaviour of the gauge theory by studying the 4-branes of type IIA theory, and demonstrate that the strong coupling limit is indeed a 5+1 dimensional theory.

3 Fourbranes in Type IIA Theory

In this section we find support for the above conjectured behaviour of the gauge theory by studying the 4-branes of type IIA theory.

The world volume theory of N 4-branes is an $\mathcal{N}{=}2$ $U(N)$ gauge theory in 4+1 dimensions. It has 5 scalars in the adjoint, corresponding to the transverse fluctuations of the brane. The gauge coupling squared of this gauge theory is proportional to λ, the type IIA coupling.

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[4]Note also that their mass is proportional to R, so they are easily interpreted as particles with a definite P_{11} [1].

[5] See, however, the discussion below regarding possible anomalies.

In M-theory this 4-brane is the M-theory 5-brane wrapped around X_{11}, which is of length λ. At strong coupling we see the 11th dimension decompactify, and more relevant to our discussion, the gauge theory itself develops an extra dimension. In fact this configuration gives more information about the gauge theory: at strong coupling the theory becomes the world-volume theory of N coincident M-theory 5-branes, with a chiral (2,0) SUSY in 5+1 dimensions. This is a non-trivial theory and it would be interesting to find in it more structure related to compactification of M-theory to seven dimensions. The description of this theory as a limit of a lower dimensional SYM theory might be useful since in the SYM theory there is no gauge invariant distinction between the tensor multiplets describing the 5-branes (Cartan generators), and the tensionless strings that connect the coincident 5-branes.

The theory in 5+1 dimensions has the required $SL(5, Z)$ as a classical symmetry. Since the theory is chiral this symmetry can be anomalous. It was shown, in fact, that the M-theory's fivebrane world-volume theory has gravitational anomalies that can be canceled by anomaly inflow from the embedding space [13]. Since this mechanism has no obvious interpretation in the present context, it is not clear to us how the $SL(5, Z)$ symmetry survives quantum corrections. This puzzle merits, in our opinion, further investigation.

The states on the 4-brane that carry momentum the extra dimension are, as disscused above, zero size instantons, which are identified in this context as the 0-branes of type IIA string theory. It is known that there exist a unique bound state at threshold of n such 0-branes and the 4-brane for every n [4]. As the mass of these states is proportional to $\frac{n}{g^2}$, they are naturally interpreted as Kaluza-Klein modes. As we take the limit of strong coupling the 4-brane becomes a 5-brane, thus recovering the M theory description.

The existence of these bound states of n 4-branes and N zero branes has a somewhat simpler proof, since it is T-dual to the configuration of N 4-branes and n 0-branes, a configuration for which the existence of a unique bound state at threshold was established in [4].

Therefore we conclude that our assumptions in the previous section about the behaviour of the 4+1 dimensional theory at strong coupling have a strong evidence in string theory [6]. Due to possible anomalies, however, we can demonstrate the existence of the

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[6]The emergence of a sixth dimensional general covariance and chiral SUSY was disscussed recently [10] for a 4+1 dimensional theory with 16 supersymmetries.

U-duality only as a classical symmetry in the Matrix theory.

4   Conclusions

As we compactify Matrix theory on more and more dimensions, more states have to be considered in the theory, all of which decouple in the flat 11-dimensional limit. First we have to add open strings connecting the 0-branes and wound around the compact dimensions. They add dimensions to the base space of the gauge theory, according to the T-duality of type II string theory. As we compactify more than 2 dimensions there are also (closed) membranes wrapped around the compact dimensions. They are discovered in the the gauge theory as states (torons) that become light at strong coupling. As we compactify more than four dimensions there are also the longtitudal fivebranes, wrapped around the compact dimensions, that have to be considered. In this paper we have shown that these states manifest themselves in the creation of an additional dimension in the Matrix theory's base space.

As we compactify yet more dimensions we expect new phenomena to occur. In particular we expect to discover the effect of wrapped transverse fivebranes addeed to the gauge theory. The guide of the U-duality appearing as a symmetry of the base space of the gauge theory, combined with some S-duality, might be useful in discovering the appropriate Matrix theory description of these compactifications (and the missing transverse fivebranes). We hope to report on work on these topics in the near future.

Appendix

For the sake of completeness we repeat here the arguments of [6] in the case of compactification on a 4-torus. We take then the spacetime parameters to be R and L_a and the gauge theory's parameters to be S_a and g^2.

The energy of a singly wound string, that is a membrane wound around the direction L_a and the 11th direction, is :

$$E = \frac{L_a R}{(l_p)^3}. \tag{4.4}$$

This is to be equated to the quantum of a momentum mode in the S_a direction, which is

$\frac{2\pi}{S_a}$, therefore:

$$S_a = \frac{2\pi(l_p)^3}{L_a R}. \tag{4.5}$$

The homogenous modes of the gauge field have a term in the action:

$$\frac{S_1 S_2 S_3 S_4}{2g^2}(\dot{A}_a)^2 \tag{4.6}$$

which leads to energy quantum $\frac{(S_a)^2 g^2}{2S_1 S_2 S_3 S_4}$. This is to be compared with the energy of a single 0-brane. The quantum of transverse momentum is $\frac{1}{L_a}$ and the quantum of 11th dimension momentum is $\frac{1}{R}$. Therefore the energy of the 0-brane is:

$$E = \frac{(P_a)^2}{2P_{11}} = \frac{R}{2(L_a)^2} \tag{4.7}$$

This yields:

$$\frac{(S_a)^2 g^2}{S_1 S_2 S_3 S_4} = \frac{R}{(L_a)^2}. \tag{4.8}$$

These two relations give:

$$g^2 = \frac{(2\pi)^2 (l_p)^6}{L_1 L_2 L_3 L_4 R}. \tag{4.9}$$

As a consistency check we calculate the energy of a 5-brane completely wound around the 4-torus and the 11th dimension,it is:

$$E = \frac{L_1 L_2 L_3 L_4 R}{(l_p)^6}. \tag{4.10}$$

This is exactly the mass of a single instanton $\frac{4\pi^2}{g^2}$, as claimed above.

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